NAVAL POSTGRADUATE SCHOOL

Monterey, California

A SURVEY OF METHODS OF
TEACHING MATHEMATICS

by

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

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ABSTRACT:

A number of nontraditional methods of teaching mathematics are studied. The methods are compared by listing their advantages, disadvantages and cost.

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1. Introduction

Cost effectiveness is becoming an increasingly important item whenever graduate education for Naval officers is discussed. For this reason it was decided to look at various methods of teaching mathematics. Most of these methods are what might be called "nontraditional", that is, different from the traditional lecture method.

These new methods are designed for self-study situations and off-campus locations. They thus would have some appeal to the student who wishes to educate himself in mathematics thereby shortening his stay at the Naval Postgraduate School. Since mathematics is an important part of a student's education at NPS, it makes sense that it would be to the student's advantage to reach a certain level of mathematical ability prior to coming to NPS.

It was known that there are available today a great variety of audio and video tapes, filmstrips and various printed materials. While it was not feasible to examine all such available materials, those that have the greatest potential for Navy use were studied and are covered in this report.

2. The Open University Materials

In recent years perhaps no educational experiment has evoked such wide interest as Great Britain's Open University chartered in 1969. In seeking to educate thousands of students at their convenience the Open University concept features radio and television programs produced by the British Broadcasting Corporation (BBC), specially written materials by teams of specialists, correspondence materials and a minimum of residence requirements.

The Open University's concept of mathematics teaching is somewhat unique. For example, the first-year course, "Mathematics: A Foundation Course (M100)" was written by a team of approximately 20 people. Each person had to justify any topic he wanted to include. The topic had to be relevant, help to build concepts the students need and fit into the main theme of the course which was mapping.

The main aim of the first course is to explain not only what mathematics does -- that is, the various ways in which mathematics can be of use in other fields -- but also what mathematics is. It seeks to provide an insight into the fundamental principles on which all branches of mathematics are based and to give the student a picture of mathematics today, both as a coherent subject in its own right and also as it serves to develop and explain other subjects.

Since the theme of the course is mapping, it is to be expected that various topics that illustrate mappings are presented. In this respect alone the course differs from the traditional "first" course in mathematics.

Following are the texts and films for the first course.

1. Text: Functions
   Films: A General Introduction to the Foundation Course
   Setting Things Up

2. Text: Errors and Accuracy
   Film: Errors that Die

3. Text: Operations and Morphisms
   Film: Anatomy of Analogy

4. Text: Finite Differences
   Film: Approximating Functions

5. Text: Inequalities
   Film: Intersecting Sets

6. Text: Sequences and Limits
   Film: What is a Limit?

7. Text: Computing I
   Film: The Structure of a Computer

8. Text: Integration I
   Film: Areas Under Curves

9. Text: Logic I: Boolean Algebra
   Films: Signposting the Course
   Logic I

10. Text: Differentiation I
    Film: The Study of Change

11. Text: Integration II
    Film: The Fundamental Theorem of Calculus

12. Text: Sequences and Limits II
    Film: Hunting the Sine

13. Text: Differentiation II
    Film: On the Surface

14. Text: Statistics and Probability I
    Film: Statistics and Probability I

15. Text: Logic II: Proof
    Film: Logic II: Proof

16. Text: Probability and Statistics II
    Film: Statistics and Probability II

17. Text: Relations
    Film: Relations

18. Text: Computing II
    Film: Towards a Computing System

19. Text: Probability and Statistics III
    Film: Statistics and Probability III

20. Text: Linear Algebra I
    Film: Vector Spaces

21. Text: Linear Algebra II
    Film: Error Correcting Codes

22. Text: Differential Equations I
    Film: See How They Grow

23. Text: Linear Algebra III
    Film: Ideas in Common

24. Text: Complex Numbers I
    Film: An Algebra of Number Pairs

25. Text: Linear Algebra IV
    Film: Linear Algebra IV

26. Text: Complex Numbers II
    Film: Complex Mappings

27. Text: Groups I
    Film: Groups I

28. Text: Differential Equations II
    Film: Differential Equations II

29. Text: Groups II
    Films: The Kernel
           Groups II

30. Text: Number Systems
    Film: Playing with Numbers

31. Text: Topology
    Film: Topology

32. Text: Mathematical Structures
    Films: An End or a Beginning
           Using the Computer Terminal

The films in the above series are all black and white, 16mm and run for approximately 25 minutes. The texts are paperback editions averaging 50 pages each.

A second course, "Linear Mathematics (M201)" is also available. As the first course it, too, runs for one year. This course shows how some of the ideas introduced in the foundations course can be developed into a body of mathematics that is interesting both in itself and because of its range of applications. The unifying concept of this course is the idea of a linear vector space. It is suitable both for those who wish to specialize in mathematics and those who have more general interests, especially if these include technology or science.

The course is mainly based on two books, "An Introduction to Linear Analysis" by Kreider, Kuller, Ostberg, and Perkins and "Linear Algebra and Matrix Theory" by Nering. Throughout the course various computational aspects of topics are discussed and opportunities are given for using the computer terminals. The following topics are included.

1. Text: Vector Spaces/Linear Transformations
   Films: What is Dimension?
   The Dimension Theorem

2. Text: Hermite Normal Form
   Film: Hermite Normal Form

3. Text: Differential Equations I
   Film: Growth and Decay

4. Text: Determinants and Eigenvalues
   Film: Invariant Subspaces

5. Text: Introduction to Numerical Mathematics: Recurrence Relations
   Film: Numerical Mathematics

6. Text: Numerical Solutions of Simultaneous Algebraic Equations
   Film: Sense and Nonsense with Linear Mathematics

7. Text: Differential Equations II: Homogeneous Equations
   Film: Linear Differential Equations

8. Text: Jordan Normal Form: Differential Equations III: Non-homogeneous Equations
   Films: Normal Forms
   Particular Solutions

9. Text: Linear Functionals and Duality/Systems of Differential Equations
   Films: Dual Spaces
   Systems of Differential Equations

10. Text: Bilinear and Quadratic Forms
    Film: Quadratic Forms

11. Text: Affine Geometry and Convex Cones
    Film: Dual Cones

12. Text: Euclidean Spaces I: Inner Products
    Film: Lengths and Angles

13. Text: Linear Programming
    Films: Linear Programming
    Linear Programming Theory

14. Text: Least Squares Approximation
    Film: Approximations

15. Text: Convergence in Euclidean Spaces
    Film: Convergence

16. Text: Numerical Solution of Differential Equations: Fourier Series
    Films: Numerical Solutions of Differential Equations
    Fourier Series

17. Text: The Wave Equation/Orthogonal and Symmetric Transformation
    Films: Oscillations of a Heavy Spring
    Rigid Transformation

18. Text: Boundary Value Problems
    Film: Boundary Value Problems

19. Text: Chebyshev Approximation
    Film: Chebyshev Polynomials

20. Text: The Theory of Games
    Film: Games Theory

21. Text: Laplace Transforms
    Film: Laplace Transforms

22. Text: Numerical Solution of Eigenvalue Problems
    Film: Numerical Eigenvalues

23. Text: Differential Equations II
    Film: Predicting Oscillations

24. Text: The Heat Conduction Equation
    Film: $\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial u}{\partial t}$

25. Text: Existence and Uniqueness Theorem for Differential Equations
    Film: The Existence and Uniqueness Theorem

With one exception (no. 16, Fourier Series) the films are black and white, 16 mm and run for approximately 25 minutes. The Fourier Series film is in color. The texts are paperback editions with an average of 55 pages in each.

In both courses emphasis is placed on the fact that texts should be studied before viewing the films, that mathematics is a language containing many symbols and hence must be read slowly. It was also emphasized that the manipulative processes of mathematics would not be stressed.

2.1 Cost

In the United States the Open University materials are distributed by

Open University Department
Harper and Row Publishers, Inc.
10 East 53rd Street
New York, NY 10022
Phone: (212) 593-7020

Black and white films cost $125 each and color films cost $275 each.

For the first foundations course the films cost $4250 and the textbooks cost $87.45. For the second course the films cost $4025 and the textbooks cost $77.05. While many students could view the same film and, in fact, copies of the films could be made, the cost of textbooks for each student is somewhat excessive.

2.2 Advantages

This section and the next are based on viewing a selected number of films. These were as follows:

First course	1.	A General Introduction to the Foundation Course Setting Things Up
	3.	Anatomy of Analogy
	10.	The Study of Change
	12.	Hunting the Sine
	14.	Statistics and Probability I
	21.	Error Detecting Codes
Second course	5.	Numerical Mathematics
	12.	Lengths and Angles

The film "Numerical Mathematics" featured a "guest" speaker, Professor Leslie Fox who spoke most of the 25 minutes. In all the other films, however, there were two lecturers who alternated about every five minutes. This procedure does much to keep the films from becoming boring. Moreover, the lecturers appear to be using a teleprompter so that the number of errors and hesitations is an absolute minimum.

All the visuals have been professionally made and they appear to be mounted with magnets so that the lecturers can change them or move them about more easily. Occasionally pieces of laboratory equipment are introduced in order to illustrate a physical situation.

Perhaps the most outstanding aspect of the films is the use of computer graphics. It is amazing to watch the changes in the zeros and in the maxima and minima of a polynomial as the coefficients are being changed. It is also most instructive to watch how the sine function can be approximated by a Maclaurin's series as more and more terms are being added.

The pacing of the films is excellent; never do they appear to be hurried. Interesting things are brought in for comparison and motivation. For example, there is a visit to an airplane factory, to a machine shop, and to a pianist who is playing an arrangement (analog) of a Beethoven symphony.

It is evident that the team of writers together with the directors and producers at BBC has produced a cohesive, professional series. The occasional use of guest lecturers is commendable since it gives students additional insight into how researchers think and perform.

In these films we are spared the usual theme music at the beginning of each film. After listening to the same tune a half-dozen times one can become, at least subconsciously, annoyed. We are also spared the long list of credits as only the minimum mention is made of the lecturers and this is done as the lecture begins.

2.3 Disadvantages

The excessive cost of textbooks has already been mentioned in Section 2.1. There is no way to eliminate this by using standard textbooks since the terminology and notation are tied to the films. The films, on the other hand, are important because they provide the motivation and the illustrative examples which clarify the theory.

Because of the unifying themes of the two courses it would be very difficult to pick out individual films for viewing, as excellent and instructive as some of them are. The courses are best used as a complete series just as they were designed.

Although some of the British accents and clipped speech are somewhat annoying, one can get accustomed to these. Somewhat more serious is saying "maths" for "math", "naught" for "zero", "π by 6" for "$\pi/6$", "echelon" and "dash" for "prime", etc. Also strange is the interchange of the decimal point and the dot used for multiplication. Finally, references to "Ernie's Lottery" and football (soccer) lose something to American ears.

3. The MIT Self-Study Materials

The Center for Advanced Engineering Study at the Massachusetts Institute of Technology (MIT) has developed a number of "packaged" subjects for practicing engineers, industrial scientists, and technical managers who wish to continue their professional education on a self-study basis. These subjects range from "refresher" courses like Calculus Revisited to graduate-level subjects like Random Processes.

The traditional classroom lectures have been captured on videotape or film, and the reading assignments, problem sets, and quizzes are provided in the form of study guides, texts, and supplementary notes. Since the lock-step of the usual "live" classroom has been eliminated, these self-study materials can be used with a great deal of flexibility to match the backgrounds, learning pace, and work schedules of a wide variety of students.

All the lectures were recorded in the Center's classroom studio on two-inch quadruplex videotape masters using broadcast quality cameras and recording equipment. They are available for distribution on 16mm sound film and in a wide variety of videotape formats.

The study guide which leads the student through the various materials in the package, is much more than a set of assignments. This guide also includes photographs of the blackboards taken at the end of each lecture, problems, quizzes, and a complete set of problem and quiz solutions.

A typical lesson is divided into several steps. The lesson begins by introducing the topic to be covered, assigning the lecture to be watched, and summarizing the key concepts of the lesson. It then assigns a section of the text to be read. Following this are four types of problems. The first are

solved problems for which both the problem statement and solution are provided. The student is expected to read these carefully before trying the rest of the problems on his own. The next category are the required problems which the student should work without consulting the solutions in the back of the guide. These are followed by optional problems which provide additional practice in varying applications. And, finally, there are remedial problems to be worked by students experiencing difficulty in a particular area.

Texts are standard publications which are specially selected for each course.

The following mathematics courses are available.

1. Calculus Revisited, Part 1; Calculus of a Single Variable. H. I. Gross, Senior Lecturer, MIT. Consists of 38 lectures, pretest, study guides, lecture notes, supplementary notes, text.

2. Calculus Revisited, Part 2; Calculus of Several Variables. H. I. Gross, Senior Lecturer, MIT. Consists of 26 lectures, study guides, supplementary notes, text.

3. Calculus Revisited, Part 3; An Introduction to Complex Variables, Differential Equations, and Linear Algebra. H. I. Gross, Senior Lecturer, MIT. Consists of 20 lectures, study guides, supplementary notes, text.

   Note - The text for the above three courses is "Calculus and Analytic Geometry" by G. B. Thomas, Jr., 4th edition, Addison-Wesley, 1968.

4. Probability. H. L. Van Trees, Professor of Electrical Engineering, MIT. Consists of 49 lectures, pretest, study guides, lecture notes and the text, "Probability and Random Processes" by W. B. Davenport, Jr., McGraw-Hill.

5. Random Processes. H. L. Van Trees, Professor of Electrical Engineering, MIT. Consists of 47 lectures, study guides and the text, which is the same as for the Probability course above.

6. An Introduction to Experimentation. E. Rabinowicz, Professor of Mechanical Engineering, MIT. Consists of 14 lectures, study guide, and text.

3.1 Cost

The cost of the lectures and notes for Probability is typical of the cost of these MIT materials.

The complete set of 49 lectures can be rented for a period of nine months. The price per student includes a complete set of printed material which need not be returned. It does not, however, include copies of the text.

10 - 19 students	$150.00 per student
20 - 49 students	$130.00 per student
50 - 99 students	$120.00 per student
More than 100 students	$110.00 per student

Alternatively, the complete set of 49 lectures can be leased for the life of the film or tape. The lease price is $12,500 and this includes 100 sets of the printed material but does not include texts.

Educational institutions are entitled to a 15% discount on all of the above items. Further information may be obtained from the following:

Mr. Arthur J. Collias
Manager, Self-Study Subject Distribution
Massachusetts Institute of Technology
Center for Advanced Engineering Study
Cambridge, MA 02139

3.2 Advantages

Professor H. I. Gross, the lecturer for the Calculus films, has a dynamic personality which comes across well on the tape. He is apparently speaking extemporaneously but there are surprisingly few errors and hesitations. His voice modulation is excellent and one does not tire of his presentation even after a 35-minute tape. He has a sense of humor but uses it sparingly and wisely.

The study materials and problems have been carefully prepared and there are many problems whose solutions are presented in detail. The lecturer has excellent penmanship and has most of the material written on chalkboards at the beginning of the lecture. In this way only occasionally does he have to write something and this is usually done for extra emphasis. One of the great advantages he has is that he is left-handed without being awkward. Thus he is able to point to material without obscuring it with his body and he can also move from left to right during the lecture in a natural way.

There is no theme music and each tape begins and ends in a simple manner. The quality of the tapes is excellent and the camera does a fine job of following the action and zooming in on the correct equations.

3.3 Disadvantages

As mentioned in Section 3.1 the cost of the films and materials is excessive. There is some question whether the personality of the calculus lecturer can maintain the student's interest over the span of eighty-four thirty-minute lectures. Some variety may be desirable in such a long series.

It should also be mentioned that apparently there is no use made of computer graphics or, in fact, anything but the chalkboard. Hence the lectures can be called examples of non-traditional teaching only in that they are on tape and can be repeated by a student.

4. Teaching Machines

A teaching machine may be defined as any audio or visual or audio-visual piece of equipment which provides individualized instruction to one or more students. Ideally the pace of instruction should be controlled by the student and he should receive some reinforcement of his successful achievements.

The audio portion of the equipment is usually a tape cassette player with the tape running at 1 7/8 inches per second. These are usually two-track tapes with a running time of approximately 30 minutes per track. The student receives the audio signal through ear phones so that he can effectively shut out extraneous noise and at the same time not disturb other students.

Cued to the audio portion is a video display. This may be in the form of 35 mm slides, filmstrips or super 8 mm stills or movies. The pictures are advanced either automatically by means of a signal on the audio tape or by the student when he is ready for the next visual display. Some machines have a means for the student to respond to questions. For example, three buttons at the bottom of the screen allow the use of multiple choice questions. If the correct button is depressed, the video display automatically advances to the next position. If an incorrect button is depressed, there is an annoying buzz and there is no advance of the video. The student reevaluates his answer to the question and tries another button.

With the increased use of self-study materials it is natural that the hardware for these presentations should undergo rapid changes in a short period. This has resulted in some problems involving compatibility of equipment. Some manufacturers, in their haste to produce equipment for which there is great demand, have modified existing machines or produced low-quality machines.

It appears that these perturbations of hardware designs may converge to some good, standard, tested products by 1975. Meanwhile potential purchasers of teaching machines are urged to proceed with caution.

4.1 Dorsett Educational Systems, Inc.

Somewhat typical of the many low-cost machines being produced is the Dorsett V90, Audio-visual Teaching Machine. This is a Viewlex machine modified by the Dorsett firm to provide three button responses for the student. The audio portion is a cassette tape and the video is a 35 mm filmstrip.

Dorsett also provides software for their machine. For example, they have a program called, "Introduction to Elementary Statistics" which consists of sixteen sections each one approximately 30 minutes in length. Following are the titles of the various sections.

1. Frequency Distributions
2. Graphical Representations of Frequency Distributions
3. Measures of Central Tendency
4. Measures of Dispersion
5. Probability I
6. Probability II
7. Probability Distributions
8. Binomial Distribution
9. Poisson Distribution
10. Normal Distribution I
11. Normal Distribution II
12. Other Probability Distributions
13. Tests of Hypotheses I
14. Tests of Hypotheses II
15. Tests of Hypotheses III
16. Review of Statistics

This program was prepared by Dr. B. L. Foote of the College of Engineering at the University of Oklahoma. Tapes and filmstrips were prepared by Dorsett. These copyrighted materials may be obtained from

Dorsett Educational Systems, Inc.
Goldsby Airport
Norman, Oklahoma 73069
Mail address: Box 1226
Phone: (405) 321-0000

There are a number of criticisms that can be made of the Dorsett machine and the Statistics program. These are listed below.

a. Some of the films are out of focus when they come into view and the focusing adjustment will not bring these in. This is true, for example, of nos. 16, 17 and 18 on ST-3.

b. In other cases the focus is off from one side to the other. This means that the two sides of the film must be focused separately.

c. The film advance mechanism is not precise enough. In many cases, the wording at the top or at the bottom is obscured and manual adjustment must be made.

d. There are typographical errors on the films. For example, no. 28 on ST-8 has

$$P \left(\begin{matrix} \text{r is written 1 standard} \\ \text{dev. from the mean} \end{matrix} \right)$$

Here the word "written" appears in place of "within."

e. Some slides are missing. For example, the one before no. 13 on ST-9 is missing.

f. It is a nuisance to rewind the filmstrips and there is the constant problem of keeping fingerprints and dirt off the film.

g. It is possible to advance the film with the on-off switch rather than by giving the correct response to the question asked.

h. All tapes begin with, "This program has been prepared by the persons listed here" and there is automatic advance not to a list of names but to the first slide in the series. These opening remarks are appropriate only for tape no. 1 but they appear on all 16 tapes.

i. The tapes were all recorded by the same voice and this gets to be monotonous after a time. Moreover, the speech is hurried and this is unnecessary because there is plenty of tape remaining in each cassette.

j. One must remember to advance the tape to its end after completion of one side. This is necessary so that when the cassette is turned over, it will begin at the correct place.

k. In summary, the fact that the program contains errors and does not run in a precise way causes the student to constantly fiddle with the various adjustments. This and errors of conceptualization tend to confuse the learner.

4.2 Charles Beseler Co.

In contrast to the Dorsett machine is the Charles Beseler Company's "CUE/SEE." This is a super 8 mm sound projection/viewing system. The sound originates in a standard cassette tape player running at 1 7/8 inches per second. There are various cueing signals on the tape to control the visual material format.

The visual material is contained in a super 8 mm standard Technicolor continuous loop cartridge. This cartridge contains 50 feet of tape which is equivalent to 3600 frames. The signals on the audio channel permit the control of film speeds from single frame to 24 frames per second. There is

also a fast forward speed of approximately 80 feet per second. A frame can be held indefinitely and there is no defocusing or light loss.

There is a built-in 6" x 8" screen in the unit which is ideal for individualized and small group learning. The picture intensity is such that viewing can be done in a fairly well lighted room. Alternatively a rear gate can be opened and the picture can be projected on a wall screen for presentation to a larger audience. This, however, requires a darkened room for best results.

The "freeze-frame" capability of the machine allows one frame to be shown while the sound track continues as long as desired. Speeds of 6, 12, 18(normal) and 24 frames per second can also be obtained. This kind of flexibility is important in teaching mathematics where a computer-generated curve is to be shown or where dynamic presentations of limits, convergence, stability, curve fitting, etc. are desired.

At present no software is available for teaching mathematics. The CUE/SEE is available from

Charles Beseler Co.
8 Fernwood Road
Florham Park, New Jersey 07932
Telephone: (201) 822-1000

4.3 3M Co.

The 3M Brand Model 625 Playback/Recorder combines a 2" x 2" photographic slide and a 3M Brand audio sound disc to create a one-piece audio visual information packet. Each slide has its own individual sound message associated with it. The program can be reversed, moved ahead or repeated in place -- all with total foolproof synchronization.

Each slide of a program is manually controlled so that the student has total control of the program at all times. A tray containing 36 slides can be used with the machine. Earphones may be used and a rear projection screen can be fitted to the machine. An ordinary screen can also be used but the room must not be well lighted.

Each Sound-on-Slide frame can accommodate any slide transparency mounted in a 2" x 2" format using cardboard, plastic, metal and glass and having a thickness of 0.100" or less. Each slide frame also contains its own individual magnetic recording disc capable of holding up to 30 seconds of sound. Discs may be erased and reused indefinitely.

The Sound-on-Film device is quite versatile and has a place in teaching problem solving, for example. The solution to a problem can be revealed piecemeal so that the student has the opportunity to try to solve the problem by himself between "revelations."

Other uses include the presentation of single-subject material which is not included in a textbook. For example, some texts in partial differential equations do not discuss the characteristic ray method of solving one-dimensional wave equations. A carefully prepared slide and sound series on this subject could be very helpful to a student.

5. NPS Video Tapes

During the past seven years a number of video tapes have been produced at NPS. The original motivation for these was to relieve the live instructor either a part or all of the time from student contact. This in turn was necessitated by a shortage of instructors.

There was an attempt at one time to use a tape prepared at the University of California at Davis for teaching FORTRAN. It was found, however, that the computer and procedures referred to did not agree with those at NPS and students were understandably confused. Thus it became a necessity to prepare tapes in the area of programming languages.

In addition to fulfilling the original objectives for which the tapes were prepared, there were a number of important by-products. Preparation of material for video taping invariably resulted in the design of a better course and also improved the instructor's presentations and techniques in the classroom. Relatively inexperienced instructors could learn much from watching a completed video tape. Not only do they see the good points of the performing instructor but they also see his faults.

5.1 Mathematics for Management

In 1968 Professor R. E. Gaskell prepared a series of videotapes for MA 2300, Mathematics for Management. This course is designed to provide the mathematical basis for modern managerial tools and techniques. It includes a review of algebra, systems of linear equations and linear inequalities, introductory material from linear programming, vectors and matrices, and a brief survey of differential and integral calculus.

These video tapes were used to present the main lecture material to a class of approximately 120 students two or three times a week. The class was then divided into 5 segments and each segment met two or three times a week with an instructor. In these smaller classes it was possible to answer questions, solve problems and do some remedial work.

As the course content changed the tapes gradually became outdated and their use was discontinued. In fall 1973, however, Professor Gaskell revised the tapes, using a transparent chalkboard. This project is not complete at this date.

5.2 FORTRAN Language

Since large numbers of students need to learn FORTRAN, it is natural to have a course in this language put on tape. Such a course is now available once or twice each quarter on the Dial Retrieval System (DRS) in the NPS Reference Library. Here the students sit in study carrels and use earphones and a small TV monitor to learn the course. The tapes are loops and may be called out at any time during the hours the Library is open. It may be necessary to repeat a tape in case the student calls for it while it is being shown on demand by someone else.

The present FORTRAN series is the third one made at NPS. It features Lee G. Litzler as instructor. The tapes are of various lengths and students are encouraged to use the computer facilities in between presentation of various topics. There is also available a set of notes and a textbook. The notes are by Mr. Litzler and R. M. Hanna and are entitled, "CS 0110, Introduction to Computers; Introduction to FORTRAN IV; Intermediate FORTRAN IV."

Two texts are used, one for the first part of the course entitled, "Information Processing" by Marilyn Bohn, published by SRA. The text for the FORTRAN proper is "Programming with FORTRAN IV" by B. S. Gottfried published by Quantum.

5.3 COBOL Language

The course CS 0113, COBOL Programming is also available on the DRS once or twice each quarter. Instructing the course on video tape is John J. Budway. Assignments and general instructions are contained in a student notebook entitled "Introduction to COBOL Programming" written by Mr. Budway.

"Standard COBOL" by Mike Murach, published by SRA is the textbook for the course. The acronym COBOL comes from "A Computer Business-Oriented Language."

5.4 Calculus Review

Many NPS students have been away from calculus for varying amounts of time. When they enter they need a review of calculus which is usually supplied by a four-hour course, MA 1100, Calculus Review. This is a fast-moving course covering functions of one variable, limits, derivatives, continuity, indefinite and definite integrals, transcendental functions, Taylor's theorem, vectors in two and three dimensions, functions of severable variables, partial derivatives and multiple integration.

In fall 1970 Professor R. W. Preisendorfer prepared a series of calculus lectures for MA 1100. The objective was to have students watch 2 or 3 tapes a week and then have 1 or 2 meetings with an instructor.

Following are the titles of the 31 fifty-minute lectures in this series.

1. The Beginning: Archimedes' Squaring of the Parabola
2. Archimedes' Alternate Approach to the Parabola and Fermat's Generalization
3. The Tangent Problem as Solved by Euclid and Newton
4. From the Parabola to the Polynomial
5. The Fertile Trinity: Composite, Inverse, and Implicit Functions
6. From Polynomials to Algebraic Functions
7. The Calculus Takes Form
8. From the Circle to the Calculus of Sines and Cosines
9. From the Circle to the Other Trig Functions and their Inverses
10. From the Hyperbola to the Logarithm
11. From the Logarithm to the Exponential
12. Some Natural Origins of the Exponential
13. Complex Numbers in the Calculus
14. The Complex Exponential: Tool par Excellence
15. Differentials in One Dimension
16. Substitution Method of Integration
17. Integration by Parts
18. Integrating Rational Functions by Partial Fractions
19. Setting Up and Solving Simple Geometric Differential Equations
20. Setting Up and Solving Simple Physical Differential Equations
21. The General First Order Linear Differential Equation
22. A Model of Biological Population Growth
23. Simple Harmonic Motion
24. Damped Simple Harmonic Motion
25. Partial Differentiation: First Order Relations
26. Partial Differentiation: Second Order Relations
27. Gradients and Moving Derivatives
28. Multiple Integration
29. Series Expansions of Functions
30. Maxima and Minima
31. Least Square Curve-Fitting Technique

These video tapes were tried for three quarters. It was found that the material was presented in a historical and philosophical perspective. This appealed to the better students who perhaps did not even need the review. The average student did not seem to be impressed by the lecturer's pedagogy.

Although Professor Preisendorfer was an excellent teacher his voice lacked sufficient modulation. He also wrote most of the material on a transparent chalkboard and this tended to slow the action down. Since most of the course was prepared during one quarter it is understandable that the lecture notes are not as complete as they should be. In some cases the exercises are too difficult for most students and there are not enough routine exercises in some cases.

These calculus films are still available and are shown on the DRS occasionally for the benefit of students who are interested in a specific topic.

6. Summary and Conclusions

From the standpoint of production the Open University materials are excellent. The smoothness of the film presentations and the use of computer graphics and other props is outstanding. For a series featuring a single lecturer at a chalkboard the MIT "Calculus Revisited" is probably as good a series as can be made using this format.

Individualized teaching machines are available in a wide range of models and prices. The greatest drawback to these at present is the paucity of software for teaching mathematics. Much needs to be done in this area before individualized teaching of mathematics can be called a reality.

Tapes prepared at NPS suffer from an overabundance of theme music and credits. Teams of instructors, rather than individuals, should be encouraged to prepare video tapes.

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A number of nontraditional methods of teaching mathematics are studied. The methods are compared by listing their advantages, disadvantages and cost.

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